SEL UNIVERSITY
POWER THROUGH KNOWLEDGE

Trigonometric Identities Example

Video Script

We can use trigonometry to calculate the instantaneous power of a single phase ac circuit in steady state conditions as shown in this figure.

The magnitude of V and I are presented as the maximum magnitude divided by √2 (which is the rms [root-mean-square] value); therefore, we must multiply by √2 in order to have the maximum value again.

We have an equation for our voltage, shown here.

$$v(t) = \sqrt{2}V_{rms}\cos(\omega t)$$

It is assumed that the current lags the voltage by an angle of φ degrees, giving us this equation, where φ is the current angle compared to the voltage angle.

$$i(t) = \sqrt{2}I_{rms}\cos(\omega t - \varphi)$$

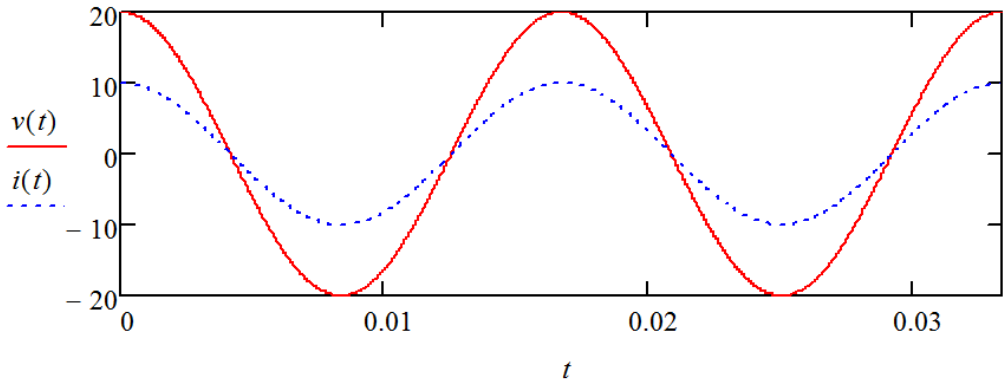
Note that in the voltage equation, we have $\sqrt{2}V_{rms}$. We know that this is equal to our V_{max}, which we can sub in. The same can be done with the current.

To help visualize these equations, we will plot them.

We can start by defining φ as 0° because we have a resistive load and the change in that angle is 0. We can assign arbitrary values for the V_{max}, say 20 volts, and the I_{max}, 10 amperes.

With this, we are given waveforms that look like this:

$$v(t) = V_{max}\cos(\omega t) \qquad i(t) = I_{max}\cos(\omega t - \varphi)$$



We see that the voltage shows a cosine waveform that has a magnitude of the V_{max} and an average value of 0.

The current also has a cosine waveform that is in phase with the voltage waveform. Similarly, we see a magnitude of the I_{max} and an average value of 0.

The instantaneous power is obtained by direct multiplication of the two sinusoidal functions representing the voltage and the current.

$$p(t) = v(t) \cdot i(t)$$

If we plot this new equation, we get an average value that is the average power. The peak value is the same as multiplying our max current and voltage values together. Because we are using 20 volts and 10 amperes, we see a peak value of 200. We have a double frequency form, which means it is alternating twice as fast as the current and voltage waveforms.

Moving back to the equation, we can bring in our equations for voltage and current and simplify. The √2 · √2 = 2. We can then multiply our V_{rms} and I_{rms}, and finally the cosine of ωt times the cosine of $\omega t - \varphi$.

We can now use trigonometric identities to turn this equation into more useful information.

If we define $\alpha = \omega t$ and $\beta = \omega t - \varphi$, then look back at our list of trigonometric identities to find an appropriate equation, we can take the value of the trigonometric identity and substitute it for the corresponding part of our equation. The result looks like this:

$$p(t) = 2V_{rms}I_{rms}\frac{1}{2}\left[\cos(\omega t - (\omega t - \varphi)) + \cos(\omega t + (\omega t - \varphi))\right]$$

We defined our values for α and β earlier, so we can substitute those for each instance of α and β and start to simplify the equation.

Starting within the brackets, we can bring down our cosine. We have $\omega t - \omega t$, which leaves us with 0. We are left with $-(-\varphi)$ which equals a positive φ. We can bring down the next cosine and work within the second set of parentheses. $\omega t + \omega t = 2\omega t$, and we have a $-\varphi$. Going back to the start of the equation, we have 2 times ½, which equals 1, so we will just bring down our V_{rms} I_{rms}. We can bring down our plus sign, then multiply our V_{rms} I_{rms} to the second portion of our equation, leaving us with the following equation:

$$p(t) = V_{rms}I_{rms}\cos(\varphi) + V_{rms}I_{rms}\cos(2\omega t - \varphi)$$

We will look back at the trigonometric identities to see if we can simplify the equation any further.

If we create a new definition for α ($2\omega t$) and β (φ), we see that we can substitute in this second trigonometric identity. We can replace the α and β with our values of $2\omega t$ and φ, then start to simplify the equation.

Working left to right, we see that we cannot simplify the first portion of the equation anymore, so we will leave that as is. The next step is to multiply our V_{rms} I_{rms} to the cosine of $2\omega t$ times the cosine of φ. We will add in the next portion of the equation, V_{rms} I_{rms} times the sine of $2\omega t$ times the sine of φ.

The result of this equation is a function with three terms.

1. The first term is equal to $V_{rms}I_{rms}\cos(\varphi)$ and is constant, meaning it does not depend on time.
2. The second term is a perfect sinusoid, but at a frequency equal to 2ω. The magnitude is proportional to the cosine of angle φ.
3. The third term is a perfect sinusoid also at a frequency equal to 2ω. The magnitude of this term is proportional to the sine of angle φ.

Using trigonometric identities, we managed to simplify the equations until we could get to some useful information. Looking at our plotted equations, we can learn a little more about our findings.

If the impedance of the circuit were a perfect resistor (where the angle φ is zero), the current and the voltage would be in phase as shown here.

Also in the case of pure resistance, the instantaneous power is always larger than or equal to 0.

In every other case, where φ is not zero, the instantaneous power assumes negative values during certain parts of the period. If we change the angle to 45°, the values dip below 0 here.

Notice that the magnitudes have not changed at all for our voltages and currents. The voltage is still a max of 20 volts and the current is still at 10 amperes. What you do see is a change in the angle difference between the voltage and the current. Because the current lags the voltage by the angle of φ, the current is no longer in phase with the voltage. It is this change in the current that affects the average instantaneous power. Note that it is no longer centered around 100.